The fractal Mandelbrot set and the shaping of the 3D fractal

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Annotation: The theory of fractals is applicable for creating the real objects and for constructing the independent elements of the framework or the whole structure of the building. Innovative program was developed to research the shaping of the 3D fractal. The object of the study is 3D Mandelbrot fractal. External structures of various powers of 3D fractal were modeled and researched. New terminology has been developed which can be applied to 3D fractals. The power factor shows the ratio of the work of external forces to the total external. The concept of forming 3D fractal can be used in the design of unique buildings and structures.
Keywords: fractal, fractal geometry, Mandelbrot set, 3D fractal, fractal structure, finite element method.

Fractals are expressed in the algorithms and sets of the mathematical operations. The fractal geometry of Mandelbrot studies nonsmooth, rough, foamy objects as well as objects capped with pores, cracks and holes [1-5].

The theory of fractals is used in the fractal analysis of the nanomodified composite microstructure [6,7] and in fractal analysis of effect of air void on freeze–thaw resistance of concrete [8, 9]. The possible field of application this theory in civil engineering is quite extensive. Fractal theory is applicable for creating the real objects and for constructing the independent elements of the framework or the whole structure of the building [10-12].

The object of the study is a 3D Mandelbrot fractal. The structure Mandelbulb can be perceived as a natural object. 3D fractals are common in human's lives, their forms are complete and understandable, at the same time the forms are in constant. The model of the structure of the 3D fractal allows to conduct research of the different frameworks.

Innovative program «External structure of the fractal set» was developed to create an external structure of the fractal [13-15]. The purpose of the algorithm is to determine the points of the fractal surface and then create a finite element model consists of shells. Topological data of the modeled fractal structure exported to the SCAD software [16]. External surface of the structure is approximated by isoparametric triangular finite elements with six degrees of freedom in the node. The input data for the calculation were the physical and mechanical characteristics of titanium applied to the elements of the fractal structure. The initial radius of the sphere for the evolution of the structure is 50 m. The models were developed with fixed number of initial points of the first iteration surface for all exploring powers.

Power of fractal or quasi-volume is an analogue of the concept of local dimension of a fractal set.

To analyze the fractal structure, the authors adopted the following terminology, which is valid for structures with power above 3.

The exostructure is the external structure of the three-dimension set consist of exobase, surface of f-quarks and crown (Fig.1).

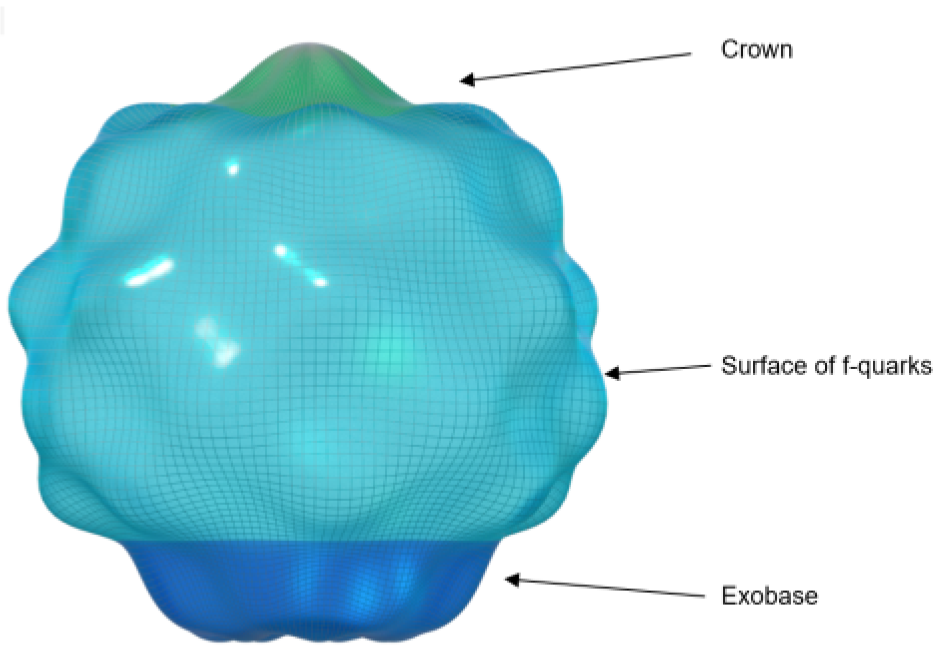

Fig. 1. – The exostructure

A quark is a type of elementary particle and a fundamental constituent of matter in physical theory. On the base of this analogy, authors introduce the notion of f-quark as a unit of the exostructure. F-quarks are classified in depending on their form in the structure of fractal: quasielliptic and quasishperical f-quarks.

The exobase is the bottom support part of the exostructure. The reference points of exobase are the reference points of quasielliptic f-quarks, which form funnel at the center by joining.

The surface of f-quarks is the middle part of the exostructure and consist of quasishperical f-quarks which have homogeneous surface distribution.

The crown is the top part of the exostructure and consist of quasielliptic f-quarks.

For power 4, the exostructure is asymmetric and heterogeneous with respect vertical axis. Evolution of the surface of f-quarks occurs with considerable fluctuations by volume. The boundary of the transition of the exobase to the surface of f-quarks is implicit. The crown occupies ¼ of the exostructure (Fig.2a). For power 5, the exostructure is symmetrical with respect vertical and horizontal axes. The crown of the exostructure repeats the exobase completely, the quasielliptic f-quarks form funnel in the both cases. F-quarks have a homogeneous distribution by surface (Fig.2b).

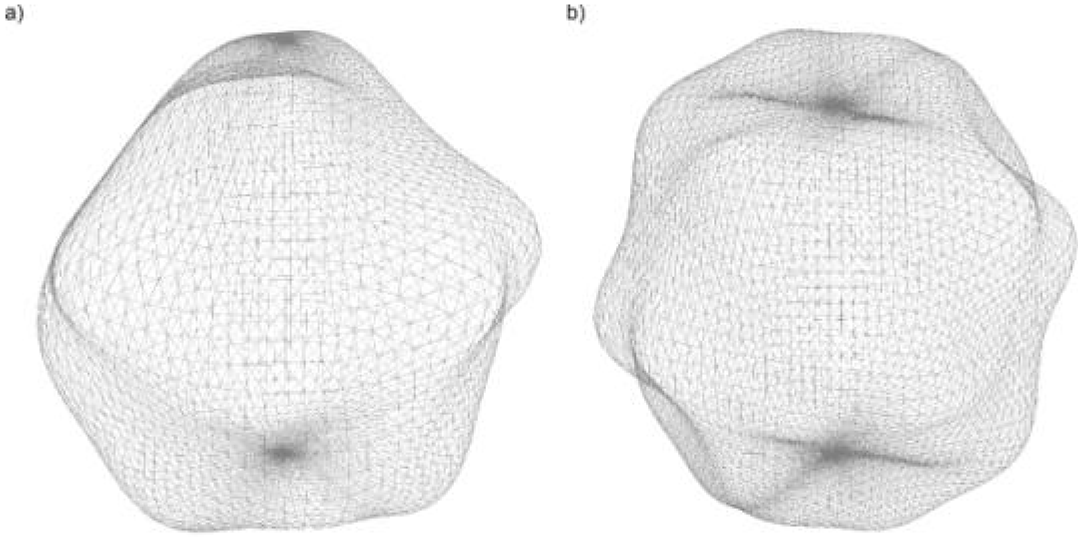

Fig. 1. – The exostructure: a) power 4; b) power 5

The exostructure of the power 8 has a pronounced exobase consisting of seven supporting quasielliptic f-quarks, which form funnel (Fig. 3a). The evolution of the fractal occurs relative to the central vertical axis. Each level of surface of f-quarks consist of the seven quasispherical f-quarks. Specialty of the even power of fractal is its crown, which consist of the quasielliptic f-quarks forming the top. The exostructure of the odd power 9 is absolutely symmetrical, it means that the shape of the crown coincides with the shape of the exobase (Fig. 3b). They are formed by the quasielliptic f-quarks, which form funnel.

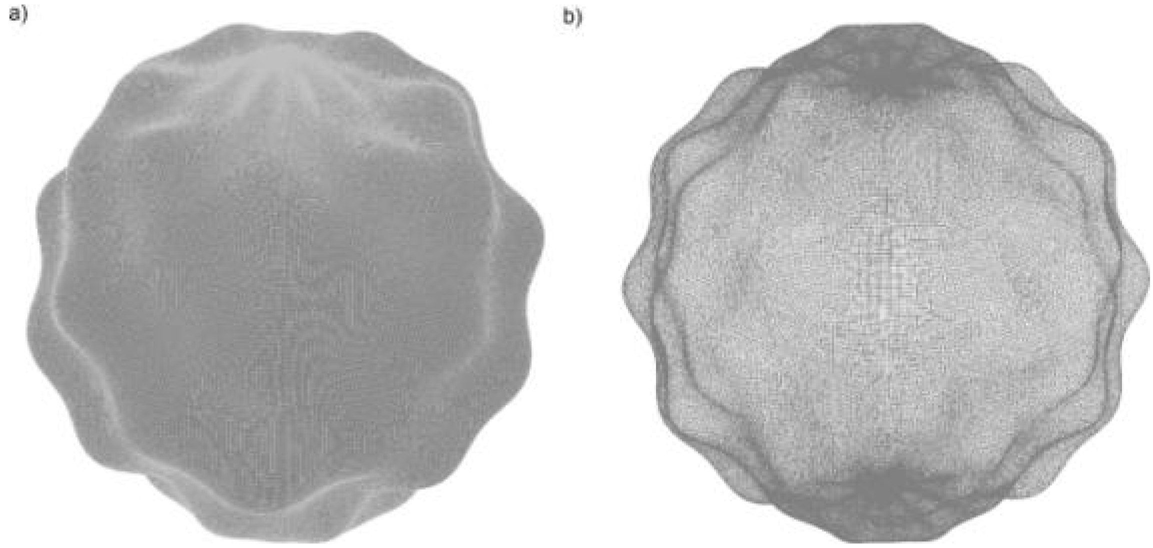

Fig.2. – The exostructure: a) power 8; b) power 9

Dependence of the shaping of structures can be clearly seen when considering powers more than 8. The results of shaping low power exostructures are significantly different from each other. For impact analysis of even and odd power of the fractal on the shape creation process, consider the cross sections in different planes. Figure 4 depicts the horizontal (a) and the frontal (b) sections of fractal surface of the power 8.

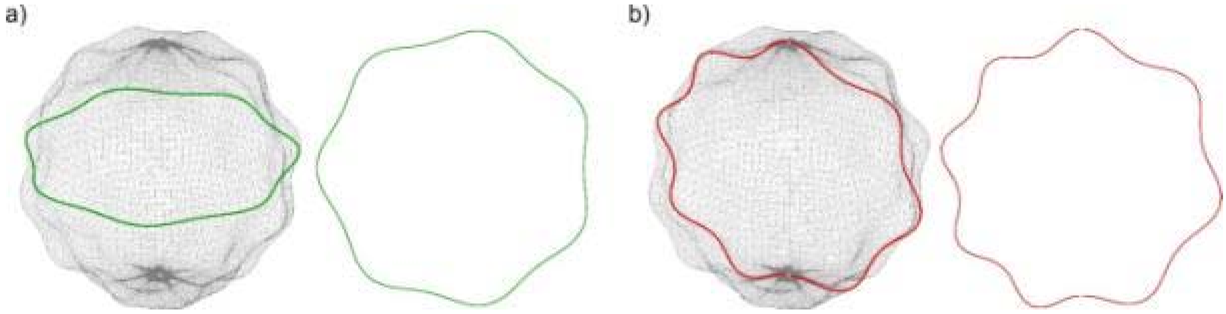

Fig. 4. Sections of the exostructure power 8: a) horizontal; b) frontal

The projections of f-quarks have homogeneous distribution in the horizontal section. The frontal section has a symmetry breaking relative to the equatorial line of the fractal, it means that f-quarks of the surface develop heterogeneously.

Figure 5 depicts the horizontal (a) and the frontal (b) sections of surface of the power 9. The evolution of the f-quarks of the middle surface, exobase and crown is ideally homogeneous on all considered planes with respect to any axes.

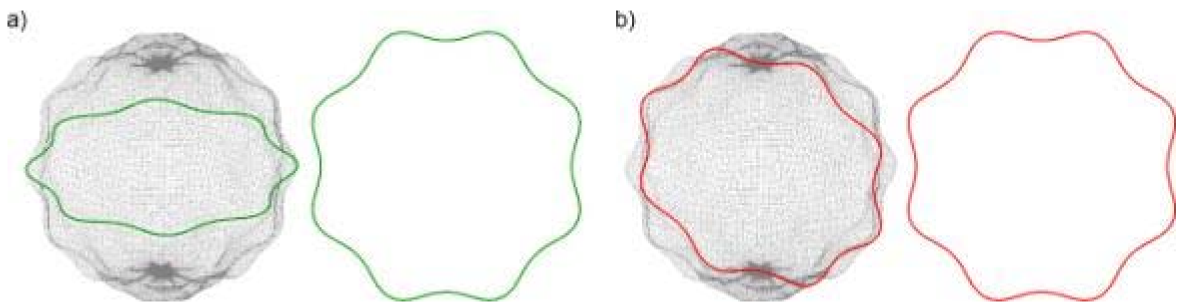

Fig. 5. – Sections of the exostructure power 9: a) horizontal; b) frontal

For the structures of odd power more than 9, crown and exobase are absolutely identical. The funnel of the crown simplifies the shape of the structure. Therefore, odd power is not considered in the future.

Determination of stress-strain state of exostructure was made by SCAD software with using finite element method [17]. Design models of the fractals were calculated on the influence of its own weight. In the numerical experiment of the exostructure shaping was investigated the even powers of fractals from 6 to 44. All models had the same number of nodes 253270, the triangular finite elements 505680, the power of system of linear equations 1517040.

The structure of the power 6 is not symmetric, the crown is strongly pronounced, also the exobase occupies the third part of the exostructure volume. The maximum radius of the location of the surface points reaches 60.7 m.

According to the authors' opinion, for the first time fractal structure acquires its original and familiar form in power 8. The exostructure of the power 8 was analyzed previously and this structure will be taken as a fundamental in subsequent study. The maximum radius of the location of the surface points reaches 57.7 m.

Considering further exostructures, we note that the crown and exobase become more detailed with the increasing powers, but occupy a smaller volume of the exostructure in constant to surface of f-quarks (Fig. 6).

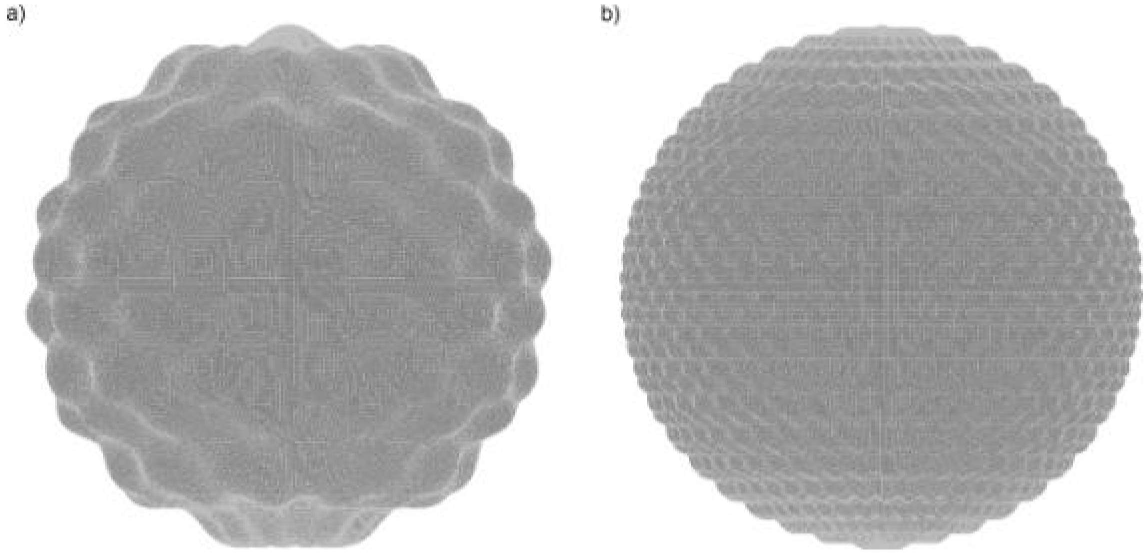

Fig. 6. – The exostructure, plane X0Y: a) power 14; b) power 44

Increasing power leads to more intensive distribution of f-quarks at the surface, the maximum radius tends to the original radius 50 m. The analysis of exostructure can be conducted by using the power coefficient of the exostructure.

Figure 7 shows a diagram of the change the power coefficient of the exostructure. For the complete compare of the external structure of fractal set was researched the spherical surface with the same input data. Power coefficient of the sphere conditionally corresponds to power 0 in a diagram.

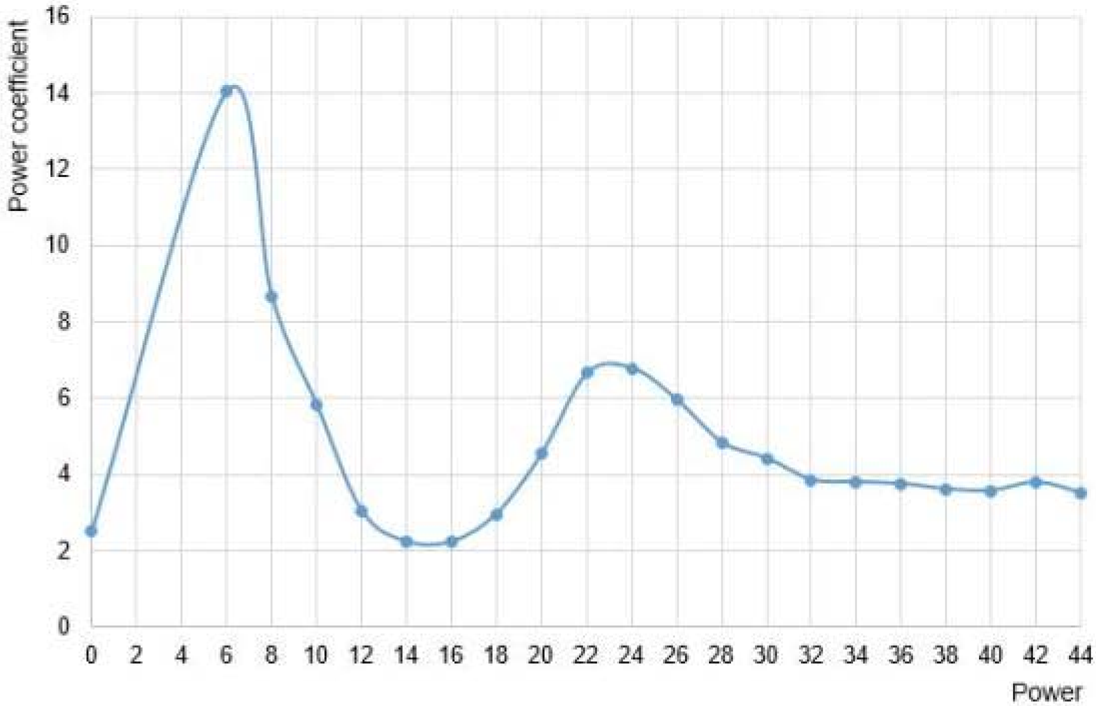

Fig.7. – Diagram of the change the power coefficient of the exostructure

For structure power 6, the power coefficient has extreme value 14.04. Then the curve of the diagram decreases nonlinearly and reach the minimum value 2.23, which one belong to powers 14 and 16. This means the possible advantage of the selected exostructures. After power 16 the diagram increases and reaches a new extreme value 6.78, which one belong to power 24. The changing of power coefficient has an undulating character, with every new extreme value less than the previous. The sphere has value of the power coefficient nearly 2.52, which is more than the minimum power coefficient value 2.23 of the exostructure power 14. As a result, the powers 14 and 16 are the most efficient.

References

1. Mandelbrot, B.B. The Fractal Geometry of Nature. San Francisco: 1982. 462 p.

2. Mandel'brot B. Fraktaly i khaos. Mnozhestvo Mandel'brota i drugie chudesa [Fractals and Chaos: The Mandelbrot Set and Beyond]. Izhevsk: NITs «Regulyarnaya i khaoticheskaya dinamika», 2009. 392 p.

3. H.-O. Peitgen, P. H. Richter. The beauty of fractals. Springer-Verlag: Heidelberg, 1986. 184 p.

4. Morozov A.D. Vvedenie v teoriyu fraktalov [Introduction to the theory of fractals]. M. Izhevsk: Institut komp'yuternykh issledovaniy, 2006. 162 p.

5. Voloshin A.V. Ob jestetike fraktalov i fraktal'nosti iskusstva [About the aesthetics of fractals and fractal art]. V kn.: Sinergeticheskaja paradigma. Progress-Tradicija, 2002. 495 p.

6. Smirnov V. A., Korolev E. V., Danilov A. M. Fractal analysis of the nanomodified composite microstructure. Nanotechnologies in Construction: A Scientific Internet-Journal, Moscow, CNT «NanoStroitelstvo». 2011, Vol. 3, No.5, pp. 77– 86.

7. Inozemtcev A.S. Srednyaya plotnost i poristost vysokoprochnykh legkikh betonov [Average density and porosity of high-strength lightweight concrete]. Magazine of Civil Engineering. 2014. No.7(51). Pp. 31–37).

8. Shanshan J., Jinxi Z., Baoshan H. Fractal analysis of effect of air void on freeze–thaw resistance of concrete. Construction and Building Materials. 2013. No.47. Pp. 126–130.

9. Pertseva O.N., Nilolskiy S.G. Optional reference method to determine frost resistance of concrete. Magazine of Civil Engineering. 2014. No.3(47). Pp. 71–76.

10. Jencks Ch. The New Paradigm in Architecture, (seventh edition of The Language of Post-Modern Architecture), Yale University Press, London, New Haven, 2002. 228 p

11. Kravchenko G. Modeling the External Structure of a Fractals. 2017 IOP Conference Series: Earth and Environmental Science. IOP Publishing, 2017. V. 90. № 1. P. 012100.

12. Kravchenko G.M., Vasil'ev S.E., Pudanova L.I. Parametricheskaya arkhitektura [Parametric architecture]. Kontseptsii ustoychivogo razvitiya nauki v sovremennykh usloviyakh: Sbornik statey Mezhdunarodnoy nauchno-prakticheskoy konferentsii [The concept of sustainable development of science in modern conditions: Collection of papers of International scientific-practical conference]. Ufa, 2017, pp. 264 – 267.

13. Jencks Ch. The New Paradigm in Architecture, (seventh edition of The Language of Post-Modern Architecture), Yale University Press, London, New Haven, 2002. 228 p.

14. Kravchenko G.M., Vasil'ev S.E., Pudanova L.I. Inženernyj vestnik Dona (Rus), 2016, №4. URL: ivdon.ru/ru/magazine/archive/n4y2016/3930

15. Kravchenko G.M., Vasil'ev S.E., Pudanova L.I. 3D modelirovanie fraktala: svidetel'stvo o gosudarstvennoy registratsii programm dlya EVM [3D modeling of fractal: the certificate of state registration of computer programs] №2017610058, 09.01.2017.

16. Karpilovskij V.S. SCAD Office. Vychislitel'nyj kompleks SCAD [SCAD Office. Computing complex SCAD]. M.: IZdatel’stvo ABC, 2007. 590 p.

17. Kravchenko G.M., Trufanova E.V., Borisov S.V., Kostenko S.S. Inženernyj vestnik Dona (Rus), 2016, №1. URL: ivdon.ru/ru/magazine/archive/n1y2016/3494